

FS – 19 / 15-16

Mathematics

Paper – I

Time : 3 hours

Full Marks : 200

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from
Section – A and Q. No. 5 from Section – B
which are compulsory and any **three** of
the remaining questions, selecting
at least **one** from each Section.*

SECTION – A

1. Answer any **five** of the following : $8 \times 5 = 40$

(a) Find an orthogonal matrix P for which
 $P^{-1}AP$ is a diagonal matrix where

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is a matrix.

(b) Determine whether in the following two cases W is subspace of the vector space V or not ?

(i) $W = \{f : f(5) = 2 - f(7)\}$, if V is vector space of all functions from real field \mathbb{R} to \mathbb{R}

(ii) $W = \{(a, b, c) ; a + b + c = 0\}$, where $V = \mathbb{R}^3$ is a vector space

(c) Using $\varepsilon - \delta$ definition, find the limit of $f(x, y) = x^2 - 2y$ as $x \rightarrow 2$ and $y \rightarrow 1$.

(d) Define and trace Gamma function of n that is $\Gamma(n)$ for positive and negative values in the domain $-3 \leq n \leq +3$. Also find the value of $\Gamma\left(-\frac{3}{2}\right)$.

(e) Find the equation of a conic in polar coordinates and discuss that in case the eccentricity is less than unity it will be an ellipse.

(f) Trace the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2}$ accurately, and name it.

2. (a) If A and B are finite dimensional subspaces of a vector space V, then show that A + B is finite dimensional and $\dim(A + B) = \dim(A) + \dim(B) - \dim(A \cap B)$. 20

(b) Prove that intersection of two subspaces of a vector space is also a vector subspace. 20

3. (a) Let $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & \text{Otherwise} \end{cases}$

Show whether the function is continuous at (0, 0) or not? Also show whether the two first order partial derivatives of the function exist or not at (0, 0)? 13

(b) Let $U = x^2y$, $x^5 + y = t$, $x^2 + y^3 = t^2$, then find $\frac{dU}{dt}$. 13

(c) Evaluate the following : 7+7 = 14

(i) $\int_0^1 \int_{\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x^2 + y^2) dx dy$ by changing to polar coordinates

(ii) $\int_0^1 \int_{\sqrt{x}}^1 \exp \frac{x}{y} dx dy$ by changing the order of integration

4. (a) Trace the conic $3(3x - 2y + 4)^2 + 2(2x + 3y - 5)^2 = 39$. 14
- (b) Find the equation of sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z = 8$, $x + y + z = 3$ as its great circle. 13
- (c) Find the equations of lines of intersection of the plane $3x + 4y + z = 0$ with the cone $15x^2 - 32y^2 - 7z^2 = 0$. 13

SECTION - B

5. Answer any five of the following : 8×5 = 40
- (a) Write a homogeneous linear third order ordinary differential equation with variable coefficients whose ordinary point is $x = 0$ and (i) a boundary value problem and (ii) an initial value problem associated with it.

- (b) Find the particular integral of the ordinary differential equation $y'' + 4y = \sec 2x$.
- (c) In a central orbit at its one point (r, θ) law of central force is inversely proportional to (r^3) . Find the pedal equation of the central orbit.
- (d) Write down the equation of Poinsot's central axis for the system of two forces $(1, -1, 2)$ and $(2, 3, 4)$ acting at points $(2, -1, 1)$ and $(4, 3, 2)$ respectively.
- (e) Derive gradient of a scalar function in cylindrical coordinate system.
- (f) Determine the unit vector perpendicular to the plane containing the two vectors $2\vec{i} - 6\vec{j} - 3\vec{k}$ and $4\vec{i} + 3\vec{j} - \vec{k}$.

6. Solve the following four ordinary differential equations : 10×4 = 40

(a)
$$\frac{dy}{dx} = \frac{y^2 + 2x^2y}{xy - 2x^3}$$

- (b) Find the general solution and singular solution of $p^3 + 8y^2 - 4xyp = 0$, where $p = (dy / dx)$.
- (c) $(xD)^3y + 3(xD)^2y + (xD)y + y = x \log x$, where $D = (d / dx)$.
- (d) $y'' + 9y = \tan 3x$ using the method of variation of parameters.
7. (a) Discuss the stability of a rigid body placed at the top of a fixed rigid body and displaced slightly from rest position, the portions of both the rigid bodies are circular in the vicinity of the point of contact. 14
- (b) Find the time period and amplitude of a rectilinear simple harmonic motion in an unresistive medium in terms of velocities and accelerations at any two positions of the path. 13
- (c) A thin hollow circular cone with its base below the vertex floats completely immersed in water. Find the vertical angle of the cone. 13

8. (a) State and prove Gauss divergence theorem
of vector calculus. 14
- (b) State and prove Serret-Frenet's formule. 13
- (c) State and prove the identity for curl of cross
product of two vectors. 13



1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in the context of public administration and financial management. The text notes that without reliable records, it is difficult to track the flow of funds and ensure that resources are being used effectively and efficiently.

2. The second part of the document addresses the challenges associated with data collection and analysis. It highlights that gathering accurate and timely data can be a complex task, often requiring the coordination of multiple departments and the use of various data sources. The text also discusses the importance of ensuring the quality and integrity of the data collected, as well as the need for robust data management systems to store and analyze the information.

3. The third part of the document focuses on the role of technology in improving data management and analysis. It discusses how modern data management tools and software can help organizations streamline their data collection processes, reduce errors, and gain valuable insights from their data. The text also touches on the importance of ensuring that these technologies are used responsibly and that data privacy and security are maintained throughout the process.

4. The fourth part of the document discusses the importance of training and capacity building for staff involved in data management and analysis. It notes that having well-trained personnel is crucial for ensuring the accuracy and reliability of the data. The text suggests that organizations should invest in training programs and provide ongoing support to their staff to ensure they are equipped with the necessary skills and knowledge to handle data effectively.

5. The fifth part of the document concludes by emphasizing the overall importance of data management and analysis in decision-making. It states that data-driven insights can help organizations make more informed decisions, identify areas for improvement, and optimize their operations. The text encourages organizations to embrace a data-driven culture and to continuously monitor and improve their data management practices.